

Quantum Kernel Methods for High Dimensional Data Classification

Maharaj Manohar¹, Prakash Shanmugam², Chella Pandi³

^{1,2,3} Sri Krishna College of Engineering and Technology, Coimbatore, Tamil Nadu, India

Received Date: 20 March 2026

Revised Date: 5 April 2026

Accepted Date: 17 April 2026

Abstract

Quantum kernel methods have become a viable option for high-dimensional data classification with the computation power of quantum systems. Fundamentally, classical kernel methods (e.g., support vector machines) have been shown to be effective by mapping data into higher-dimensional, explicitly defined feature spaces where linear separability is achieved. But when data dimensionality becomes large, classical methods encounter issues of computational feasibility and scalability. Addressing these challenges, quantum kernel methods involve mapping the data onto quantum feature spaces by encoding them in terms of quantum states processed on quantum circuits. This allows for efficient representation and manipulation of complicated data distributions. In this paper, we study the theory, practice and advantages of quantum kernel methods for high-dimensional classification. It addresses quantum feature mapping methods, kernel estimation with quantum circuits, and hybrid quantum-classical learning frameworks. Empirical evidence shows that quantum kernels can improve classification performance for certain tasks, especially when classical kernels fail to capture complex correlations. The paper alludes to real-world issues, including noise, the need for a ton of qubits and hardware limitations. But all of this is beside the point: quantum kernel methods mark a major advance to realizing at least some aspects of quantum advantage in machine learning and which open up exciting possibilities for solving complex classification problems arising in real-world applications.

Keywords

Quantum Kernel Methods, Quantum Circuits, Feature Mapping, Classification of High-Dimensional Data by Support Vector Machines

Introduction

High-dimensional data classification is among the most pressing problems in contemporary machine learning, fuelled by an exponential increase of available data across domains such as healthcare, finance, image processing and scientific research. Datasets in these settings tend to have very large numbers of features, which can number in the thousands or millions of dimensions. Based on these features, we assign/pre-define data points to various categories using classification primary objective. Although classical ML algorithms perform relatively well on such tasks, their performance starts to deteriorate as the dimensionality increases. This is often known as the “curse of dimensionality” and translates into computational difficulty, data sparsely, and challenges to find meaningful patterns. Thus, creating efficient and scalable methods for high-dimensional classification is an important research topic.

Kernel methods already had a long history of success tackling high-dimensional problems with non-linear classifiers. Support vector machines (SVMs) are one such area that is gaining traction owing to its strong theoretical underpinning and practical success. This is accomplished using kernel functions which allow SVMs to implicitly map data into higher-dimensional feature spaces and compute the transformation without having to calculate it directly. And with this, the algorithm can find linear decision boundaries in transformed spaces of data to make it separable. It has long been known that polynomial and radial basis function (RBF) kernels are a more general framework for capturing complex relationships in data. Despite the advantages of kernel methods, classical kernels have several significant drawbacks when applied to very large or high dimensional datasets. Kernel methods become less scalable due to the growing computational cost of kernel matrices and their rapidly increasing memory requirements for available kernel-based approaches.



Beyond the calculations to perform, classical methods also tend to struggle when it comes to the correlation and structures that grow complex in advanced datasets. With increase in dimensionality, traditional kernels effectively learn the relationships between data become poor. This limitation reveals that more expressive and computationally efficient methods with the capability to handle high-dimensional data should be developed. Fortunately, breakthroughs in quantum computing provide a potential answer as they introduce fundamentally new methods for the manipulation of information.

Quantum computing relies on the principles of quantum mechanics — such as superposition and entanglement — that allow for quantum systems to provide new methods for representing and manipulating information that are impossible for classical systems. In contrast to classical bits that exist purely in one of the two states 0 or 1, quantum bits (quits) can be multiple states at once. This property enables quantum computers to perform many possibilities at the same time, which potentially gives them an edge over classical computation in terms of solving complicated computational problems. This ability is further fortified by entanglement, allowing for quits to be strongly correlated and act in unison as a whole. Such features allow for quantum computing to shine with high-dimensional data and difficult patterns.

These properties of quantum systems are applied to improve performance of classical kernel-based learning models. With this method, quantum states are prepared by parameterized quantum circuits and classical data encoded to produce a quantum feature space that could be exponentially larger than its classical version. It then uses quantum operations to measure the similarity between data points, or in other words, computing kernel values as such in this infinitely-dimensional space. With this process, quantum kernel methods become machines capable of diagnosing these complex relations and patterns. Hence, these methods can lead to a dramatic improvement in classification performance for high-dimensional settings.

Furthermore, quantum kernel methods are also built on successfully interfacing with classical machine learning frameworks. Because quantum hardware is still in its infancy, hybrid quantum-classical methods are standard. For these types of systems, the kernel values are computed using quantum circuits, with classical algorithms orchestrating the optimization and decision-making process. This hybrid approach allows for the practical use of quantum-enhanced models on existing hardware while still relying on the quantum computational advantage. It furthermore offers a variety of insights into the natural workings of classical and quantum techniques, paving new paths for their unionization and composite improvement.

Unfortunately, quantum kernel methods also have several drawbacks despite their potential. But, when it comes to quantum hardware DE coherence noise and limited quits are limiting its capabilities in terms of accuracy and scalability. Efficient data encoding and circuit design continued to be other areas of research. Overcoming these challenges is key to unlocking the power of quantum machine learning for practical use cases.

To conclude, it bridges together quantum computing with kernel methods which is a milestone in ML. Quantum kernel methods provide a promising strategy to address the inadequacy of classical approaches in classifying high-dimensional data using quantum feature spaces and hybrid computational models. This chapter discusses the drive and obstacles to this new line of inquiry, building the groundwork for several aspects of quantum-enhanced machine learning solutions.

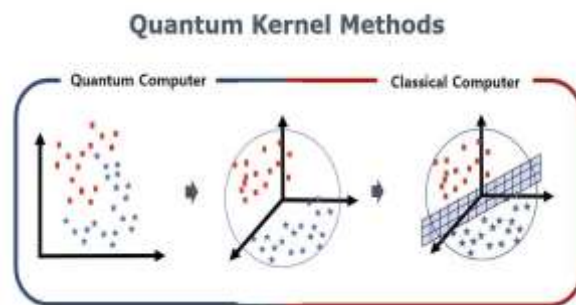


Figure 1: Quantum Kernel Method Architecture

Fundamentals of Kernel Methods

Kernel methods are one of the fundamental algorithms in machine learning, generalized for classification and regression of complex non-linear data. Importantly, kernel methods are fundamentally based on the idea of quantifying how similar two data points are. So, instead of transforming data explicitly into higher dimensional feature spaces which can be computationally expensive, kernel functions enable algorithms to compute inner products in these spaces without getting their hands dirty. This technique commonly known as the 'kernel trick' implements efficient approaches to deal with non-linear relationships without having to explicitly compute high

dimensional transformations. Consequently, kernel methods have gained great popularity as a flexible and robust set of tools in areas such as pattern recognition, bioinformatics and image analysis.

Kernel methods are widely used, a famous example of which is the support vector machines (SVM), where the goal is to find the best decision boundary separating data points within different classes. For data that is not linearly separable in its original space, kernel functions will map the data into a higher-dimensional space in which there exists a linear separator. This is what enables SVMs to perform well in complex classification tasks. Common kernel functions include the (1) linear kernel for linearly separable data, (2) polynomial kernel, which accommodates polynomial combinations of features to introduce non-linearity and (3) radial basis function (RBF) kernel, especially effective in capturing localized patterns & relationships in data.

Kernel methods are powerful tools because they can be flexible and model some of the complexity of structures present in data. With the selection of an appropriate kernel function, learning algorithm can be made to fit the data. As an example, RBF kernel is commonly found to be popular for applications where the relationship between features is highly non-linear while also polynomial kernels are useful in capturing interactions between variables. This is why kernel methods are extremely flexible and general in their applicability to tasks across the machine learning literature.

Kernel methods also have another important advantage: they are well-founded theoretically. These originate from concepts in functional analysis and optimization theory that give guarantees on generalization performance and robustness. This makes SVM to avoid over fitting, for example by using margin maximization. Maintaining this trade-off between accuracy and generalization is likely to influence the performance of kernel-based methods.

However, kernel methods also face challenges when extended to high dimensions or large-scale settings. The computational cost of kernel evaluation is one of the main restrictions. Many kernel methods involve the computation of a kernel matrix — a form of pairwise similarity measures between all data points. The kernel matrix grows quadratically with dataset size, imposing significant memory and computation overhead. As a consequence, using kernel methods on very large datasets is not possible unless we use approximation or some kind of specialized optimization methods.

Kernel methods are also not immune to the problems of parameter selection in addition to computational complexity. A kernel-based model's performance requires its kernel function and parameters like degree for polynomial kernel or bandwidth for RBF/browner. Finding the best parameters can be so cumbersome and needs cross validation which also increases the overhead on computation. In addition, when the data itself is high-dimensional, these kernels can fail to be effective and cannot effectively capture genuine patterns.

A second challenge is that classical kernel functions are not necessarily expressive enough to represent very complex data structures. Kernel methods can model non-linear relationships but may still fail to capture complex dependencies in very high dimensional data. This bottleneck has pushed the researcher to reach for the suitable methodologies that may improve feature representation power but with better computability.

In summary, kernel methods are an important aspect of modern machine learning that bypasses the computational complexity of explicit mapping to non-linear feature spaces by inferring this mapping implicitly through positive semi-definite kernels. Because of their flexibility, solid theoretical foundation and broad field of applicability, they are a powerful tool for classification and regression learning. Their shortcomings in terms of scalability, computational cost, and expressiveness point to a need for more rigorous methodologies. The face of these issues forced the emergence of quantum kernel methods to use quantum computing for overcoming limits set by classic approaches and unlocks more potent solutions for high-dimensional data classification.

Quantum Computing Basics

A. Introduction to Quantum Computing

Quantum computing is a new way of computation that has the potential for a paradigm shift in the field of computational science (Lane, 2023). Contrary to classical computers that use bits their information in 0s or 1s either way, quantum computers leverage quantum bit - or qubits - which can be all available states together. This key distinction allows quantum systems to complete some computations significantly faster, especially those with high dimensional data and complex optimization problems. Hence, various disciplines like cryptography, material science and machine learning have paid attention towards quantum computing.

B. Qubits and Superposition

The qubit is the fundamental building block of quantum computation. While classical bits store information as a state, either 0 or 1 (both not at the same time), qubits use superposition principles and can be in both stated simultaneously. A single qubit can also represent multiple possible values simultaneously, and a system of multiple qubits can represent an exponentially-determined number of states. Superposition enables quantum bits (or qubits)

to process information in parallel and is one of the driving forces that gives quantum systems their computational advantage. This feature is primarily beneficial for machine learning tasks where huge and complex datasets need to be analysed in an efficient manner.

C. Quantum Entanglement

Entanglement is another basic concept of quantum computing. Entanglement is the quit phenomenon in which two or more quits become correlated in such a way that the state of one quit is mentioned to be influenced by another, irrespective of their distance. Such a strong correlation enables quantum systems to perform cooperative operations unattainable in classical systems. Entanglement is central to the function of many quantum algorithms, allowing for, among other things, efficient sharing and processing of information between quits. For quantum models, entanglement may help to capture complex relationships among data and outperform classical counterparts.

D. Quantum Gates and Operations

Quantum Gates (like Logic Gates) Quantum gates are a fundamental part of quantum circuits, as they were in classical computing. These gates control the state of quits via operations such as rotations, phase shifts and entangling. Examples of quantum gates are the Hadamard gate (superposition), the Pauli gates (basic transformations) and CNOT gate introducing entanglement. However, in a quantum computer unlike classical gates, the quantum gates are reversible and they work on probability amplitudes rather than deterministic values. Several gates together form complex operations, so researchers can create complex algorithms.

E. Quantum Circuits and Computation

Quantum circuits are arrangements of quantum gates that act one after another on quits to perform calculations. These circuits determine how quantum information gets processed in an algorithm and mapped. In a typical quantum-circuit model, quits are initialized, then subjected to a sequence of gates before submitting a measurement procedure that collapses the quantum state to a classical output. Designing quantum circuits is important to the development of quantum algorithms, as it largely determines how efficiently and accurately anything we do with a quantum computer can be performed. Many quantum machine learning methods utilize parameterized circuits that can be trained and optimized similar to classical neural networks.

F. Measurement and Probabilistic Outcomes

Measurement is an unconventional interface of quantum computing, distinguishing it from classical computation. The quantum state of a qubit collapses into one of the basis states (usually 0 or 1) when it gets measured. The result is probabilistic due to the amplitudes of the quantum state beforehand. And this stochastic nature offers several chances and certain pitfalls, for quantum computing. Although it enables uncertainty and complex distributions to be represented, it demands several rounds of measurements when a valid result is needed. Measurement is one of the key things from quantum computing, indeed you should train first to understand it before jump on outputs of quantum algorithms.

G. Relevance to Machine Learning

Digital Quantum Computing principles, including –superposition, entanglement & quantum operations– serve as an effective basis to ramp up the current machine learning algorithms. Quantum systems offer the promise of evading many limitations of classical approaches by allowing efficient representation and processing of high-dimensional data. Since tasks like classification and clustering rely on feature mappings, similarity measures and model parameters optimization, quantum circuits can be leveraged to perform these operations. With the alignment of a major interest in quantum machine learning research, mastering these basic components is essential for harnessing increasingly sophisticated and efficient models.

H. Conclusion

In conclusion, quantum computing represents an entirely new paradigm of computing and has various advantages whenever you deal with a complex as well as high dimensional problem. These are the concepts of quits, superposition, entanglement and quantum circuits; all this enables powerful computational capabilities. These essentials are critical for using advanced applications like quantum kernel methods and hybrid quantum–classical machine learning models. These basic principles will be at the heart of intelligent systems as quantum technology continues to advance.

Quantum Feature Mapping

Quantum feature mapping is a cornerstone of quantum machine learning, especially for high-dimensional data classification using quantum kernel methods. The embedding function encodes classical data into quantum states (called an "ersatz") using parameterized quantum circuits, mapping the data to a quantum feature space. This is important because it allows quantum systems to exhibit representations of complex patterns and relationships that might be challenging or impossible to capture using classical feature mappings. No, Quantum Feature Mapping

has nothing to do with your race or gender: Quantum feature mapping is a method that takes advantage of quantum mechanical principles like (superposition and entanglement) in order to build much more expressive feature spaces thus acting as a useful pre-processing operation for classification algorithms.

Often in classical machine learning, feature mapping is applied to transform data into a space which is simpler for separation or analysis. Computing high-dimensional mappings explicitly can be computationally expensive, if not intractable for large datasets. Quantum feature mapping tackles this challenge by using quantum state preparation to implicitly embed data into exponentially large Hilbert spaces. This allows a state with only a few qubits to represent an unimaginably rich feature space, which provides classical features with an edge. This efficient access and manipulation of these high-dimensional representations is one of the core motivation of machine learning using quantum computing.

Quantum feature mapping is done through quantum circuits, where quantum feature maps are the parameterized forms shown. These circuits use classical data as input and encode it into the amplitudes or phases of quantum states. Surely, the circuit designs impact the feature-mapping quality. If the feature map is well configured then it will capture many relations and structures in data, thus gives more accurate classification. On the other hand, if a feature map is not designed correctly, it may miss important patterns thus leaving the model ineffective.

In quantum feature mapping, there are several encoding methods to choose from; each has its own advantages and drawbacks. Here, angle encoding is one of the most frequently employed techniques to encode data values as rotation angles of quantum gates. This method is quite easy to use, and also does not need as many computational resources making it apt for near-term quantum devices. We also have amplitude encoding, which allows for direct representation of data in the amplitudes of quantum states enabling high efficiency in representing large datasets with fewer qubits. On the other hand, amplitude encoding is more challenging to implement and it takes possible extra computational cost. Another approach is to use basis encoding, which directly maps the binary data onto qubit states: it is very simple but does not allow for expressively representing continuous or high-dimensional data.

The selection of an encoding method depends on various considerations like the size and nature of the dataset, available quantum hardware, and limitations imposed by the application. One example is that angle encoding typically works better for real-time or resource-constrained applications because it is relatively simple, and amplitude encoding is appropriate where it's necessary to represent a large dataset efficiently. Sometimes hybrid encoding methods are employed that combine the advantages of different techniques so as to reach a compromise between efficiency and expressiveness.

Quantum feature mapping is especially critical in quantum kernel methods, which depend on calculating the similarity between data points from their representations within a quantum feature space. Good feature mapping implies that the data will be largely separated in the transformed space, which results in an effective kernel. When the quantum feature map assumes a good shape that better separate different classes, it can result in the higher classification accuracy of the model. Feature mapping therefore becomes a central part of quantum kernel-based learning algorithms.

Quantum feature mapping has its advantages but is also plagued with issues as well. In order to create feature maps successfully, we need a solid background in quantum computer and the study of data structure. Moreover, noise and hardware constraints can influence the precision of quantum state preparation, which may in turn impact the mapping effectiveness. Dealing with these problems is a research topic, as more stable and faster encoding methods are being developed.

Overview Quantum feature mapping is a fundamental aspect enabling quantum-enhanced machine learning, as it allows us to transform classical data into concise and expressive quantum feature spaces. It permits the representation of intricate data structures, which are challenging to capture employing classical techniques, through parameterized quantum circuits and the sophistication found in encoding methods. With the development of quantum technology, it is assumed that feature mapping methods will be advanced and make quantum kernel methods achieve higher application performance in high-dimensional data classification.

Quantum Kernel Estimation

Quantum kernel estimation is one of the key pillars of quantum machine learning, as this implements the actual evaluation of similarity between data points in a quantum feature space. While classical kernel methods calculates similarity using mathematical functions whose formulation is fixed in advance, quantum kernel methods assumes that, similar to physical classifiers, similarity is relative to the distinct ways two states evolve with time based on their physical interactions. This is done by mapping the classical data to quantum states using well-designed quantum feature maps, and then measuring how much the two states overlap. This gives us a kernel value,

which indicates how similar two data points are in the adequate quantum space, which is then applied to classification algorithms like support vector machines.

To perform quantum kernel estimation, the classical data has first to be encoded into quantum states with the help of parameterized quantum circuits. Each data point maps to a unique quantum state thus embedding the input data into a high-dimensional Hilbert space. After data encoding, the inner product between pairs of quantum states is calculated by quantum circuits. That inner product, which we will call the fidelity or overlap, be the kernel value. The extent of the overlap determines how close in similarity or proximity the two states are deemed to be which is how similar their respective data points too. This approach enables quantum systems to learn complex relations and correlations between features that are difficult to represent with classical kernels.

An important benefit of quantum kernel estimation is its potential to conduct in tremendously large feature procedures without directly manipulating excessive-dimensional conversions. Mathematical functions are used in classical kernel methods to approximate such transformations but these become computationally extremely intensive with increasing dimensionality of data. Whereas classical systems are limited to low-dimensionality, quantum systems often occupy high-dimensional spaces as a matter of course due to superposition and entanglement. It allows for an efficient representation and processing of complex data structures, making quantum kernel methods a great tool for high-dimensional classification tasks. Direct estimations of quantum kernels are most often done with custom-built quantum circuits that will measure overlap between states. The swap tests are a classic way to compare 2 quantum states by entangling the two and checking your probability of feeling certain outcomes. The other approach is to create circuits that directly perform fidelity calculations using interference patterns. These techniques enable kernel values to be estimated accurately while reducing the overall number of quantum operations required. Nevertheless, the effectiveness of these methods varies depending on parameters such as circuit depth, noise and architecture.

Hybrid quantum-classical machine learning frameworks also rely on quantum kernel estimation, for example. These systems often use quantum processors for computing kernel values and classical algorithms for tasks such as optimization and classification. This hybrid method takes advantage of the benefits from both quantum and classical computers, allowing implementation on existing quantum hardware. It allows for the combination of quantum-feature spaces with classical learning algorithms, leading to better performance in applications, especially highest-dimensional and complex-data solutions for specific use cases. Though it has advantages, quantum kernel estimation faces a variety of challenges. One of the main challenges is that noise exists in quantum hardware, which can impact the precision by which kernel measurements are conducted. Noisy kernel values make the model less powerful (e.g., due to errors in quantum state preparation, gate operations, and measurement processes). Moreover, the number of qubits and coherence time in existing quantum devices restrict circuit size and complexity at present. To tackle these hurdles we need better quantum algorithms that are less impacted by noise and error mitigation approaches.

The selection of quantum feature map is one more crucial aspect that directly affects the rank of the kernel. And different kernel matrix can affect the performance of classification algorithm producing different feature maps. Therefore, choosing an appropriate feature map is crucial in order to yield the most effective result. One area of current research deals with the design of feature maps which can sharply increase the expressivity power of quantum kernels while being efficient and robust to noise.

Table 1: Comparison of Classical vs. Quantum Kernel Estimation

Aspect	Classical Kernel Methods	Quantum Kernel Methods
Feature Space	Implicit, limited by computational cost	Exponentially large quantum feature space
Computation Method	Mathematical kernel functions	Quantum state overlap (inner product)
Scalability	Limited for very high-dimensional data	Potentially scalable with quantum advantage
Complexity	High computational cost (kernel matrix computation)	Efficient representation via quantum states
Hardware Requirement	Classical computing systems	Quantum processors (NISQ devices)
Noise Sensitivity	Low	High (due to quantum noise)
Expressiveness	Depends on chosen kernel function	High due to superposition and entanglement

Overall, quantum kernel estimation is a strong method for measuring similarity in high-dimensional quantum feature spaces. Using quantum circuits to calculate inner products between the quantum states, it leads towards more expressive and potentially more efficient classification models. Although many challenges in the form

of noise and hardware limitations are still apparent, we expect that progress in quantum technology (hardware) as well as algorithm design will continue to improve the practical applicability and performance of quantum kernel methods.

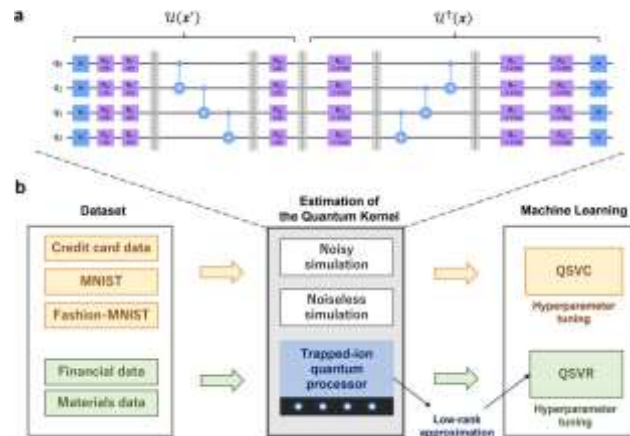


Figure 2: Quantum Kernel Estimation Workflow

Hybrid Quantum-Classical Learning Models

This is where hybrid quantum-classical learning models are a practical, yet powerful solution to integrate quantum computing into machine learning workflows. However, at this stage of quantum hardware development where we still face noise and the number of qubits is extremely limited, it is not viable to have fully quantum machine learning systems for general large-scale applications. Designed to overcome this problem, hybrid models combine quantum kernel computation with classical machine learning algorithms and allow the use of existing devices in a convenient way. Within this structure, a quantum circuit serves primarily as the feature mapping and kernel evaluator, with classical systems performing the optimization, training and decision making processes. This versatility gives researchers the opportunity to reap the benefits of quantum computing without being limited by hardware constraints.

Usually hybrid quantum-classical models are composed of two components: a quantum processing unit and a classical computation unit. The quantum part represents classical data as well, and computes kernel values by measuring the overlap of these states. The kernel values computed in this way are then fed into the classical component and used for training the usual machine learning models like support vector machines. This creates a feedback loop, where the classical part adjusts parameters and feeds fresh values to the quantum circuit. The process continues iteratively until the model gets an optimal solution. This architecture is modular, allowing flexibility and scalability for a variety of use cases.

Model — Quantum kernels: A hybrid learning model, which measures the similarity between quantum feature spaces using a distance fusion mechanism. These kernels are computed based on quantum circuits and encodes more complex relationships between data points than can be obtained with classical kernels. Classical algorithms use the resulting kernel matrix to classify or perform regression tasks. Hybrid models can therefore be better equipped to learn high-dimensional and non-linear data through the use of quantum feature spaces. Which makes them even more useful for applications where classical methods fail to grasp complicated patterns?

Though quantum circuits perform feature mapping and kernel computation, optimization and learning is performed through classical algorithms. To train the model based on the calculated kernel values, gradient descent, quadratic programming, and support vector machine optimization techniques are utilized. The stability, scalability and matured software's eco-system of classical systems bring them much suited for such tasks. Classical optimization methods incorporated into the form of hybrid-composite models guarantee an effective training procedure associated with hybrid devices and the lack of computational resources from the quantum side. The success of hybrid approaches is prominently based on this division of responsibilities into quantum and classical components.

Hybrid quantum-classical learning models combine the advantages of a classical and a quantum part, while pure quantum or purely classical approaches tend to strongly over fit. This allows for quantum-enhanced feature spaces to be amortized while still providing the robustness and efficiency of classical algorithms. This is what makes them very suitable for today's near-term hardware due to the fact that current quantum computing technology is still in its infancy. Also, Hybrid models are flexible and can be run on different quantum platforms such as superconducting and trapped-ion systems. That said there are some challenges to be faced: communication latency between quantum and classical components, noise in quantum computations yet to be addressed and better data

encoding techniques certified with an efficiency/roadmap. These challenges need to be addressed in order to improve the scalability and performance of hybrid systems.

High-Dimensional Data Challenges

Modern machine learning applications have witnessed the increasing popularity of high-dimensional data, especially in genomics, image processing, natural language processing, and financial analytics. These datasets are often known to have a dimensionality that is higher than the number of samples and impose a computational and analytical barrier. The “curse of dimensionality” is one of the greatest problems linked to high-dimensional data; this is a phenomenon in which given that number of dimensions increases, the volume grows exponentially. This leads to sparse data points which make it practically impossible for machine learning models to recognize any patterns or relationships of interest. Such sparsely makes traditional algorithms less useful and cause significant generalization performance deteriorations.

The other significant challenges of the analysis of high dimensional data are greater computational complexity. A lot of classical machine learning algorithms including kernel-based methods require operations that do not scale well with the number of features and data points. As an example, creating kernel matrices requires functioning the calculations of similarities from all pair of data points and this operation is increasingly costly when datasets become larger. This leads to longer processing time and a large amount of memory requirement, resulting in small scalability for classical approaches. In practice, these limitations hinder the applicability of conventional methods on large-scale high-dimensional data sources without using approximations or resorting to dimensionality reduction approaches.

Furthermore, over fitting is an important issue when working with high-dimensional data. If one has more features than training examples, the models may learn noise or irrelevant patterns instead of learning the underlying structure in the data. This results in low performance on unseen data, making the model less reliable. Regularization and feature selection (classical techniques) may help with over fitting but not significantly on highly-dimensional datasets. Furthermore, finding the most relevant features in such datasets can be quite challenging and also takes a long time.

Another major downside classical methods face is their inability to capture complex relationships in high-dimensional data. Kernel techniques are a type of algorithm, but they rely on kernel-choosing and hyper-parameter involving settings that can be finely-tuned to avoid over fitting in theory. Often, when the dimensionality is very high, instead of encoding complex relationships that exist on the data classical kernels perform poorly. This limitation emphasizes the requirements of having more expressive feature representations, which can capture complex structures without imposing excessive computational costs.

Quantum kernel methods have a lot of potential to help with these issues through the unique abilities of quantum computing. These methods map data to large quantum feature spaces, which can be exponentially larger than classical spaces by encoding a given classical data in quantum states. This results in more expressive representations of data, helping to capture the complexity of relationships that are hard to model using classical techniques. Additionally, quantum systems operate in parallel thanks to superposition, which could alleviate scaling problems associated with computationally expensive high-dimensional data.

Quantum kernel methods also compute similarities between data points using quantum operations, which is more efficient for some problems. Quantum circuits implicitly encode these high-dimensional transformations, thus allowing transportation of the concepts directly to quantum domain where kernel can be efficiently evaluated. This method not only enhances in scalability but also improves the performance of classification algorithm over complex dataset.

Although quantum methods have the potential advantages, they are still early stage and suffer issues of noise, hardware restrictions, etc. Such challenges can be overcome by advancing quantum technology (and vice versa), especially when it comes to improving the practicality of quantum kernel methods for applied settings.

To summarize, high-dimensional data is often problematic for classical machine learning algorithms due to increases in complexity, over fitting and reduced expressivity. Quantum kernel methods are an alternative that accommodates increased expressibility of feature spaces and computational virtue. These techniques are anticipated to be key in overcoming the limitations of classical methods and allowing us to analyze high-dimensional datasets more efficiently as quantum technology progresses.

Analysis of Quantum Kernel Methods Performance

Quantum kernel methods performance is evaluated by a multi-dimensional evaluation framework considering both classical and quantum computational aspects. Quantum kernel methods are very different from

traditional machine learning models, due to their hybrid nature, thus many metrics for evaluation have been established. The main metrics of interest are accuracy, scalability, computational cost, and resilience to noise. Together, these metrics define the performance of quantum kernel models on high-dimensional classification tasks. Furthermore, classic methods of evaluation involve comparing to classical kernel methods such as the support vector machine on radial basis function kernels. This insight serves as a comparative analysis that helps identify situations in which measurable quantum advantage can be achieved.

Accuracy is one of the most important metrics to assess quantum kernel methods, representing the number of data points classified correctly by a model. Quantum kernels utilize high-dimensional quantum feature spaces that can capture complex relationships and correlations between data points with state-of-the-art performance which cannot be directly accessed from classical kernels. For complex structured datasets or data with non-linear relationship, this usually leads to a more class accurate classification. Quantum kernel methods have been experimentally shown to be more powerful than classical models for some tasks, especially those where the data has properties that match well with quantum feature mappings. But the degree of improvement is contingent on the quantum kernel choice, data and physical system limitations.

Experimental feasibility is an important consideration in the practicality of quantum kernel methods for real-world applications. Kernel methods from the classical ML epoch are not scalable due to a quadratic explosion in kernel matrix size with data set size. Quantum kernel methods: For a fixed number of qubits, quantum operators correspond to a low number of representatives in the exponentially large feature spaces. Allows dealing with high dimensional data more efficiently. However, contemporary quantum hardware is limited in its scalability, owing to small qubit number and circuit depth. Advances in hardware capabilities due to the progress of quantum technology can further augment and scale up the applications of quantum kernel approaches while preparing them for large-scale adoption.

Performance analysis also considers computational efficiency. Quantum kernel methods: Quantum state overlaps can lead to efficiencies through implicit computation of inner products in high dimensional spaces (e.g. quantum states). This reduces the need for explicit feature transformations, which is costly in nature with classical system. In practice, though, the effectiveness of quantum methods depends on circuit architectures, measurement complexity and amounts of noise. For hybrid quantum-classical architectures, the total computation time encompasses both quantum evaluations and classical optimization steps. Although quantum circuits speed up certain computations, the efficiency of computation can be affected by communication overhead between quantum and classical parts. This interaction needs to be optimized so that practical performance gains can be achieved.

Noise Robustness: With current noisy intermediate-scale quantum (NISQ) devices, noise robustness is an important criterion to use when assessing quantum kernel methods. Ancillary-assisted quantum computations are very likely to be affected by errors due to DE coherence, imperfections in gates and statistical error of measurements. Such factors could affect the quality of kernel estimation and later, the performance of classification models. To increase robustness, a variety of related error mitigation techniques have been applied, including noise-aware circuit design and post-processing corrections. Notably, the quantum kernel methods are quite resilient to these challenges in some specific scenarios, suggesting that they could be robust at higher levels (as hardware matures).

Comparison of classical and quantum kernel methods offers a good perspective in terms of advantages and disadvantages. Classical techniques are also old, stable and efficient for a lot of applications but they are dead slow on high-dimensional data set or complex dataset. On the other hand, quantum kernel methods achieve better expressiveness and may solve certain problems more effectively. Key differences can be summarized as follows in the table below:

To sum up, quantum kernel methods shows a promise of outperforming its classical counterparts and might scale to much higher dimensionality datasets. These models can work in complicated quantum feature spaces, improving accuracy and representation of the data. Although scalability, noise problems, and impedance mismatch still pose challenges to engineering progress, the continuous development of quantum technology is likely to overcome these issues. Consequently, quantum kernel methods are expected to help raise the bar of machine learning in the near future as these techniques become critical components for applications that demand computational power and sophisticated data representation capability.

Optimisation of Quantum Kernels

Optimization is an essential part of quantum kernel methods as it provides solution to high-dimensional classification tasks. Quantum kernels offer expressive feature spaces, however their efficacy is closely tied to the optimization of the quantum circuits and classical learning ingredients which comprise them. In this context, optimization refers to the adaptation of parameters for quantum circuits, kernel estimators, and the running of

hybrid quantum-classical workflows. So quantum kernels have some prospects, But those advantages will be limited in noiseless hardware.

The quantum feature maps are usually defined in terms of parameterized quantum circuits (PQCs). Such circuits have tunable parameters (e.g. rotation angles) that determine how data is mapped to quantum states. Tuning these parameters is necessary for obtaining maximal separability between data in the quantum feature space. These parameters are typically tuned using classical optimization techniques like gradient descent or evolutionary algorithms. Other quantum-aware optimization techniques (e.g., parameter-shift rules, which allow gradient estimation directly from quantum circuits to accelerate convergence) are also available.

Kernel alignment is an optimization method that enhances high-quality quantum kernels. This involves tuning the kernel function to better suit the underlying data distribution and the classification task. The more the alignment between the kernel matrix and target labels, then it will help in better classification by model. Similarly, classical components (like regularization parameters in support vector machines) are optimized using techniques such as cross-validation and hyperparameter tuning.

Currently available quantum devices possess a considerable amount of noise, and optimization methods should integrate with hardware imperfections as well. Traditional techniques for noise-aware optimization include using circuits of smaller depth, choosing gate sequences which are robust to errors and the usage of quantum error mitigation. This lessens the effects of DE coherence, and increases the reliability of kernel estimation. Furthermore, noise patterns can be predicted by ML models and can also help to direct optimization decisions.

To summarize the optimization strategies play vital role in boosting quantum kernel methods. Leveraging classical optimization methods alongside quantum-aware methods could enhance quality, performance and robustness of the solution. Improvement in quantum hardware will continue to sweep in and advanced optimization methods will be critical to extract maximum value from quantum machine learning.

Interpretability and Explainability on Quantum Kernel Methods

The adoption of machine learning models in production setting has grown over the years, but understanding their complexity and governing decisions have become one of biggest pain point. This is particularly relevant to high-stakes domains like healthcare, finance and security where understanding how a model arrived at its decision is crucial. The use of quantum feature spaces and hybrid computational frameworks makes quantum kernel methods powerful yet increases the complexity. Consequently, understanding why these models reach certain decisions can also be an enormous obstacle. In this chapter we define interpretability of kernel methods within the quantum context and present a novel approach for enhancing model interpretability.

Similarly, quantum kernel methods work in very high-dimensional non-observable quantum feature spaces. Feature importance analysis is often accomplished through weights or gradients for classical models, but quantum states used to encode this information collapse on measurement. This makes it hard to track how features of individual inputs correlate with the final classification. Furthermore, the probabilistic foundation of quantum computing presents an additional complexity because outputs arise from measurement distributions and not deterministic calculations. All of which, significantly hampers the effectiveness of conventional Explainability techniques in quantum scenarios.

Analyzing feature importance in quantum kernel models is one direction which may enhance interpretability. You can do this by checking how changes in input data are reflected through to kernel values. Sensitivity analysis methods can be leveraged to quantify the contribution of each feature individually towards the similarity between two data points. It is possible to see which features influence the model's decisions most if you systematically change input features and analyze how classifications change. While this is far from the full picture, it does shed light on how quantum kernel models behaves.

Even quantum feature mappings can be understood using these visualization techniques. Even though quantum feature spaces are inherently high-dimensional, these spaces can be approximated in lower dimensions, by applying dimensionality reduction methods like principal component analysis (PCA). This lets researchers see how data points are distributed post-quantum transformation and whether the classes they want to separate fit well into different sections. These sorts of visualizations can give an intuitive appreciation to us about how effective the quantum kernel is and what way improvement could better be done.

Now, we know that quantum kernel methods are a kind of hybrid method. Hence, the hybrid of quantum models with classical Explainability techniques naturally leads to a feasible approach. Hybrid Outputs: The traditional tools SHAP (Shapley Additive explanations) and LIME (Local Interpretable Model-agnostic Explanations) can be amended to study the outputs of hybrid models. These methods estimate the contribution of input features to final prediction allowing us to obtain a more human-readable and understandable representation of concerned

model behavior. Combining these approaches with well-established quantum kernel methods can lead to some degree of performance-transparency trade-off.

In real-world scenarios, interpretability is crucial since decisions have to be explained and verified. With respect to healthcare, clinicians may need to know why a model predicts a certain diagnosis. Finance: Regulators need open risk assessment models since these domains often drive use cases with substantial repercussions, quantum kernel methods must therefore embed Explainability features to ensure trust within and adoption of their technology. Hence, it will be vital to consider interpretable quantum models in the context of theory advancement and applications deployment.

Explainable quantum artificial intelligence is a nascent research field, but several directions remain full of promise. This includes quantum-native Explainability techniques, advanced visualization tools and interpretable evaluation metrics. With improvements in quantum hardware and algorithms, it is predicted that there will be developing techniques to comprehend and interpret quantum types more effectively.

Figure 2: In Quantum kernel methods, interpretability and Explainability is very important for industrial applications. Although challenging due to the complexity of quantum systems, which may go beyond human reason understood in classical terms, techniques like feature analysis [5], visualization [6] and hybrid Explainability approaches that extract information from both quantum mechanics and classical machine teaching models each combine to offer several avenues towards an explanation. Enhancing the interpretability of quantum machine learning models will be paramount to building trust and ensuring successful practical deployment as research advances.

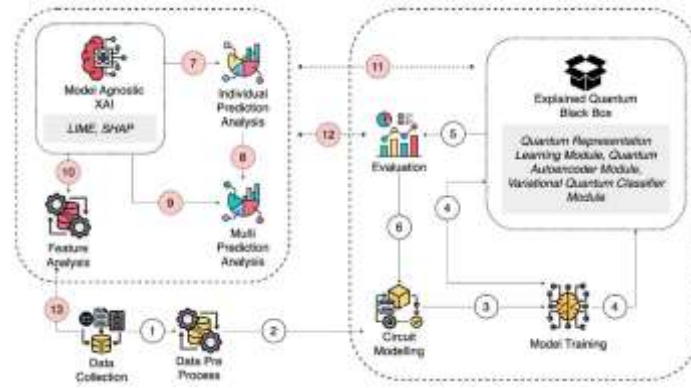


Figure 3: Explainable Quantum Kernel Workflow

Research Directions and Future Trends Of Quantum Kernel Methods

Quantum Kernel Methods for High-dimensional Data Classification is an area of fast-developing research, shaped persistently by advances in quantum computing, machine learning and hybrid computational frameworks. Although the current implementations display very encouraging results, quantum kernel methods are still far from their full potential. Future work will likely build upon existing limitations, enable improved model performance and provide more generalisation of these methods to various domains. Quantum kernel methods will likely be a pivotal ingredient to next-generation intelligent systems as quantum hardware matures and new algorithms come online.

One of the most important future directions is the building of fault-tolerant quantum computing. All existing quantum devices belong to the noisy intermediate-scale quantum (NISQ) era, where errors and DE coherence disrupt the computation. Such limitations directly influence the precision of quantum kernel estimation and, thus, the suitability of classification models. On-going progress in quantum error correction techniques and hardware stabilisation are hoped to shrink noise levels and facilitate the implementation of richer quantum circuits. Such scaling will enable quantum kernel methods to be applied to larger scale datasets and more complex applications, revealing their actual computational benefit.

Developing more efficient and expressive quantum feature maps is yet another key research domain. By definition, quantum kernel methods depend on the encoding of classical data into quantum states. Future work will centre on high reparability quantum feature spaces combined with relatively easily constructible circuits. It is also likely to provide adaptive and data-driven feature mapping techniques for models. Models can adjust the way they encode if the characteristics of their input data change dynamically. Due to these advancements, the classification accuracy would improve and quantum kernel methods would become more robust and adaptable.

Derivative of the Quantum Kernel Methods with Advanced Machine Learning Hybrid models which combine quantum kernels with deep learning architecture such as neural networks offer deeper performance by exploiting the advantages of both approaches. One such approach is where quantum kernels can be employed, to obtain high-

dimensional feature representations, processed by classical deep learning models used for classification or regression problems. Combining quantum and classical techniques can also lead to substantial performance gains in solving complicated, high-dimensional problems of interest.

Scalability is one of the main problems and a relevant research goal for the future. Although quantum systems have the potential for exponential computational power, hardware limitations and measurement cost make practical implementation unlikely at this time. To overcome these challenges, researchers are investigating methods drawn from kernel approximation, sampling optimization and distributed quantum computation more efficient quantum algorithms, coupled with better hardware architectures will lead to improved scalability as well as performance.

Quantum kernel methods are also gaining traction in real-world applications. These approaches are likely to be used in drug discovery, climate modelling, financial forecasting and even cyber security as the technology matures. As quantum kernel methods are good at dealing with high-dimensional and complex datasets, they are especially adaptable to these applications. The translation of theoretical advances to real-world solutions will depend on continued collaboration between academia and industry.

Another relevant aspect of quantum kernel methods is the necessity for standardization and benchmarking. Common foundations for evaluation frameworks, datasets, and performance metrics will allow researchers to compare diverse approaches, isolate best practices. This will help speed up development, and deployment of different quantum machine learning technologies.

Furthermore, the ethical aspects and the responsible application of quantum technologies are emerging. It also ensures that the applications of quantum computing are in harmony with humanitarian values and do not bring new threats into the world. Least unbiased, fairness, transparency and accountability for both the development and application of quantum ML systems.

Finally, quantum kernel methods have a bright future but also many challenges remain. This area will evolve as further progress is made in quantum hardware, algorithm design and hybrid learning frameworks. Finally, quantum kernel methods can represent a new research avenue to surpass current limitations and evolve towards illuminating high-dimensional data classification as well as contributing to the development of intelligent systems in general. These techniques are expected to be key drivers in the future of machine learning and computational science as research continues to advance.

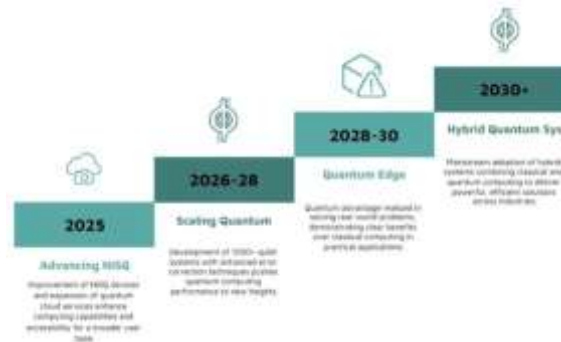


Figure 4: Evolution of Quantum Kernel Methods (Timeline View)

Conclusion

Quantum kernel methods are a ground-breaking step in machine learning to overcome many of the problems faced by traditional algorithms when it comes to high dimensional data classification. As contemporary datasets become larger, less homogeneous, and higher dimensional, many machine learning strategies face growing challenges in terms of efficiency and accuracy. Although classical kernel methods can be very efficient and powerful, they also suffer from poor scalability issues, high computational costs, and the potentially intractable nature of modelling very complex patterns in data. Quantum kernel methods provide a plausible alternative based on the advantages of quantum computing, allowing for more flexible feature spaces that are not easily accessible with classical approaches.

Quantum feature mapping, that is, the mapping of classical data into quantum states using parameterized quantum circuits, lies at the heart of quantum kernel methods. This mapping enables representations of data in high-dimensional Hilbert spaces, capable of better capturing more complex relationships and correlations. Quantum kernels offer a new approach to quantifying relationships between data points, something that is usually accomplished through similarity measures computed via quantum state overlaps. This method improves the

functionality of classification.. algorithms, especially in situations that classical kernels are not enough to reflect well the data structure.

Hybrid learning models that mix quantum and classical systems make the application of quantum kernel methods even more powerful. In these methods, quantum circuits are responsible for feature mapping and kernel computation while classical algorithms perform optimization and decision making. It combines quantum computational power with the robustness and scalability of classical systems. Consequently, hybrid models represent an intermediate point on the path from existing technological constraints towards entirely quantum machine learning systems.

The performance evaluation of quantum kernel methods provides a much wider set of applications where classical algorithm may be outperformed. Metrics such as accuracy, scalability, and parametric efficiency show promise in quantum kernels provide better solutions for high-dimensional non-linear datasets. Yet, the degree of this superiority is contingent on factors such as a feature map selection and dataset characteristics and quantum hardware quality. However these approaches are not currently a one size fits all solution compared to classical methods, indicating there is still research and fine-tuning that needs to be done.

As advantageous as they are, however, there are still number of tasks that need to be solved before quantum kernel methods can gain excessive acceptance. Today, quantum hardware is present in the noisy intermediate-scale quantum (NISQ) regime with a few dozen up to several hundred qubits, which has a limited lifetime and is readily susceptible to errors. Such limitations limit the accuracy of quantum computations and restrict circuits from being implemented with high complexity. Another open research topic is how best to design feature maps and encode data, as both can significantly impact the performance of quantum kernels.

Another important challenge is scalability. Quantum system sites provide exponential speedup in principle, but the amount of qubits available is limited by the measures required to actually realise it. Ultimately, having to perform multiple quantum measurements to approximate kernel values can introduce significant overhead and negate some of the advantages of quantum processing. Solving these problems will necessitate progress in both quantum hardware and the principles used to design quantum algorithms, for instance error mitigation strategies and compact quantum circuits.

The future of quantum kernel methods is bright as we look forward. Improvements in quantum hardware, including fault-tolerant quantum computers, will result in computational reliability and scalability improvements. Meanwhile, continuing developments in quantum algorithms and machine learning will produce more powerful yet compact models. Extending quantum kernel methods in conjunction with other new technologies, including deep learning and edge-computing solves the problems further across more domains.

Quantum kernel methods have the power to change a number of real-world applications, specifically areas that rely heavily on high-dimensional data analysis. Improved classification accuracy and the ability to discover more intricate patterns in data would only apply to fields like healthcare, finance, cyber security, scientific research. When these methods become more practical, they will start to contribute to the development of next-gen intelligent systems heavily.

Overall, quantum kernel methods provide a more powerful and novel approach to the classification of high-dimensional data, overcoming many challenges presented by classical machine learning approaches. Although a lot of work still needs to be done, the continuous progress on quantum computing and machine learning is getting us closer. With the rapidly evolving field of quantum computing, the quantum kernel methodologies are expected to be an integral part of future intelligent systems capable of unlocking new levels of performance, efficiency and insight in data-driven applications.

References

- [1] Nielsen, M.A. and Chuang, I.L., 2010. *Quantum Computation and Quantum Information*. Cambridge: Cambridge University Press.
- [2] Reskill, J., 2018. Quantum computing in the NISQ era and beyond. *Quantum*, 2, p.79.
- [3] Belmonte, J., Witter, P., Pancetta, N., Rebentrost, P., Weise, N. and Lloyd, S., 2017. Quantum machine learning. *Nature*, 549(7671), pp.195–202.
- [4] Schulz, M. and Petruccione, F., 2018. *Supervised Learning with Quantum Computers*. Cham: Springer.
- [5] Havelock, V. et al., 2019. Supervised learning with quantum-enhanced feature spaces. *Nature*, 567(7747), pp.209–212.
- [6] Rebentrost, P., Mohsen, M. and Lloyd, S., 2014. Quantum support vector machine for big data classification. *Physical Review Letters*, 113(13).
- [7] Lloyd, S., Mohsen, M. and Rebentrost, P., 2014. Quantum machine learning. *Nature Physics*, 10(9), pp.631–633.
- [8] Benedetti, M., Lloyd, E., Sack, S. and Fiorentine, M., 2019. Parameterized quantum circuits as machine learning models. *Quantum Science and Technology*, 4(4).
- [9] Schulz, M., Sweke, R. and Meyer, J.J., 2021. Effect of data encoding on quantum machine learning. *Physical Review A*, 103(3).

- [10] Abbas, A. et al., 2021. The power of quantum neural networks. *Nature Computational Science*, 1(6), pp.403–409.
- [11] Carazo, M. et al., 2021. Variational quantum algorithms. *Nature Reviews Physics*, 3(9), pp.625–644.
- [12] McLean, J.R. et al., 2018. Barren plateaus in quantum neural network training landscapes. *Nature Communications*, 9(1).
- [13] Schulz, M. and Kilogram, N., 2019. Quantum machine learning in feature Hilbert spaces. *Physical Review Letters*, 122(4).
- [14] Huang, H.Y. et al., 2021. Power of data in quantum machine learning. *Nature Communications*, 12.
- [15] Carlo, G. and Troyer, M., 2017. Solving the quantum many-body problem with machine learning. *Science*, 355(6325), pp.602–606.
- [16] Dunk, V. and Briegel, H.J., 2018. Machine learning and artificial intelligence in the quantum domain. *Reports on Progress in Physics*, 81(7).
- [17] Broughton, M. et al., 2020. Tensor Flow Quantum. *Nature*, 589.
- [18] Kilogram, N. et al., 2019. Continuous-variable quantum neural networks. *Physical Review Research*, 1(3).
- [19] Arête, F. et al., 2019. Quantum supremacy using a programmable superconducting processor. *Nature*, 574.
- [20] Mandala, A. et al., 2017. Hardware-efficient Variational quantum Eigen solver. *Nature*, 549.
- [21] Time, K., Bray, S. and Gambetta, J.M., 2017. Error mitigation for short-depth quantum circuits. *Physical Review Letters*, 119(18).
- [22] Endo, S., Benjamin, S.C. and Li, Y., 2018. Practical quantum error mitigation. *Physical Review X*, 8(3).
- [23] Li, Y. and Benjamin, S.C., 2017. Efficient Variational quantum simulator. *Physical Review X*, 7(2).
- [24] Harrow, A.W., Hassidim, A. and Lloyd, S., 2009. Quantum algorithm for linear systems of equations. *Physical Review Letters*, 103(15).
- [25] Grover, L.K., 1996. A fast quantum mechanical algorithm for database search. *STOC*.
- [26] Short, P.W., 1994. Algorithms for quantum computation. *FOCS*.
- [27] Aaronson, S., 2013. *Quantum Computing Since Democritus*. Cambridge University Press.
- [28] Cao, Y. et al., 2019. Quantum chemistry in the age of quantum computing. *Chemical Reviews*, 119(19).
- [29] Tang, E., 2019. Quantum-inspired classical algorithms. *STOC*.
- [30] Goodfellow, I., Bagnio, Y. and Carville, A., 2016. *Deep Learning*. MIT Press.
- [31] Bagnio, Y., 2013. Representation learning: A review. *IEEE TPAMI*, 35(8).
- [32] Hinton, G.E. and Salakhutdinov, R.R., 2006. Reducing dimensionality with neural networks. *Science*, 313(5786).
- [33] Krizhevsky, A., Sutskever, I. and Hinton, G.E., 2012. Image Net classification with deep CNNs. *NIPS*.
- [34] Aswan, A. et al., 2017. Attention is all you need. *Neurals*.
- [35] IBM, 2021. *Qiskit Documentation*. Available at: <https://qiskit.org>
- [36] Microsoft, 2022. Quantum Development Kit. Available at: <https://learn.microsoft.com>
- [37] Google AI Quantum, 2019. Quantum processor research. *Nature*.
- [38] NIST, 2022. *Post-Quantum Cryptography Standardization*.
- [39] IEEE, 2024. *Quantum computing research and standards*.
- [40] World Economic Forum, 2022. *Quantum computing governance principles*.
- [41] Schulz, M., 2021. Supervised quantum machine learning models. *arrive*.
- [42] Gilberto, C., Herter, M., Along, A.D., Postil, M., Brochette, A., Severing, S. and Bossing, L., 2018. Quantum machine learning: a classical perspective. *Proceedings of the Royal Society A*.
- [43] Witter, P., 2014. *Quantum Machine Learning: What Quantum Computing Means to Data Mining*. Academic Press.
- [44] Weise, N., Kapok, A. and Sore, K.M., 2014. Quantum algorithms for nearest-neighbour methods. *Quantum Information & Computation*.
- [45] Farsi, E. and Even, H., 2018. Classification with quantum neural networks. *Arrive*.
- [46] Mitral, K., Ngoro, M., Kitagawa, M. and Fuji, K., 2018. Quantum circuit learning. *Physical Review A*.
- [47] Schulz, M. and Petruccione, F., 2020. Machine learning with quantum computers. *Quantum Science and Technology*.
- [48] Meyer, J.J., Schulz, M., 2020. Information content of quantum states. *PRX Quantum*.
- [49] Benedetti, M. et al., 2020. A generative modelling approach for benchmarking quantum circuits. *NPJ Quantum Information*.
- [50] Huang, H.Y., Keung, R. and Reskill, J., 2021. Predicting many properties of a quantum system. *Nature Physics*.